The Effect of Device Modeling on a Parameter Switching Continuation Method Used in Waveform Balance Simulations

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***Abstract*— The effect of device modeling on a continuation method used to determine the large signal voltage and current waveforms of a nonlinear circuit is investigated. It is found that modeling the nonlinear element in such a way that the resulting waveform balance error function is continuously differentiable greatly enhances the performance of the algorithm.**

***Index Terms*— Continuation method, nonlinear circuit, parameter switching, waveform balance.**

I. Introduction

Waveform balance is a common method for determining the voltage and current waveforms in circuits containing nonlinear elements [1]–[3]. Many nonlinear circuits do not have unique solutions for the waveforms, and generally, one wants to find as many of the solutions as possible. Continuation methods are useful tools for finding multiple solutions on the same solution branch [2], [4], [5]. A parameter switching algorithm that gives rise to a continuation method is described in [5].

It is shown in this paper that the same method is justified by application of the implicit function theorem [8]. In this context the applicability of the algorithm is justified by a certain nonvanishing determinant and a certain smoothness of the nonlinear equations. This approach emphasizes the true local nature of the algorithm as well as the desirability of modeling the nonlinear elements in such a way that the resulting waveform balance error function is continuously differentiable. From a theoretical point of view the smoothness of the error function is only a sufficient condition. A short comparative study shows that, at least for the circuit used in the comparison, a smooth error function greatly enhances the performance of the algorithm.

II. The Algorithm

The problem considered here is one of finding the current and voltage waveforms of a circuit such as depicted in Fig. 1.

The circuit consists of a periodic independent voltage source labeled v_s, a linear element labeled Y, and a nonlinear element depicted by the diode and nonlinear capacitor. η is a scalar multiplier of the amplitude of the periodic voltage source. i_l is a function of time such that the current through the linear element Y at time t is equal to $i_l(t)$, $i_n(t)$ is the current through the nonlinear element at time t and $i_e = i_l + i_n$. The voltage source labeled v is not part of the circuit, but is inserted to enable the calculation of the function i_n and, hence, solve the waveform balance problem.

The problem that must be solved is the following. Given a periodic function v_s with period T, find a function i_n (there may be more than one) such that all circuit equations are satisfied when the current through the nonlinear device at time t is equal to $i_n(t)$.

The problem can be solved with the method of waveform balance in the following way. Assume that η is a fixed scalar. Assume that the actual waveforms are periodic with period T and can be approximated by band-limited waveforms. (If the circuit is capable of period doubling, then T should be chosen accordingly.) This implies that there exists a N such that the waveform functions, v_s, v, i_l, i_e, and i_n can be completely represented by N samples at times $t_n = nT/N$, $n = 0, 1, \cdots, N-1$. Let $\vec{v}_s$ represent the column vector $(v_s^0, \cdots, v_s^{N-1}) = (v_s(t_0), \cdots, v_s(t_{N-1}))$ and define $\vec{v}$, $\vec{i}_l$, $\vec{i}_e$, and $\vec{i}_n$ in the same way. Assume an appropriate starting value for $\vec{v}$ and adjust $\vec{v}$ until the error current $\vec{i}_e$ is zero. If $\vec{i}_e$ is zero, the voltage source labeled v may be removed from the circuit without affecting any waveforms, and $\vec{i}_n$ (representing i_n) so obtained is a solution to the circuit equations.

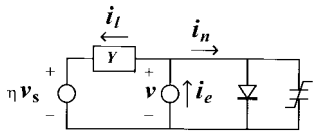

Fig. 1. Type of circuit considered.

To perform the adjustment procedure using a Newton–Raphson method a (generally nonlinear) function

$$e(\eta, \vec{v}) = (e^0(\eta, \vec{v}), \cdots, e^{N-1}(\eta, \vec{v}))$$

such that

$$i_e^k = e^k(\eta, \vec{v}), k = 0, 1, \cdots, N-1$$

as well as the Jacobian matrix

$$J = \begin{bmatrix} \dfrac{\partial e^0}{\partial v^0} & \cdots & \dfrac{\partial e^0}{\partial v^{N-1}} \\ \vdots & & \\ \dfrac{\partial e^{N-1}}{\partial v^0} & \cdots & \dfrac{\partial e^{N-1}}{\partial v^{N-1}} \end{bmatrix}$$

are calculated at each step of the iteration procedure. In particular

$$e(\eta, \vec{v}) = A \times (\vec{v} - \eta \times \vec{v}_s) + g(\vec{v}) \tag{1}$$

where A is a N by N matrix calculated from the frequency domain admittance Y of the linear element and $g = (g^0, \cdots, g^{N-1})$ is determined by the nonlinear element. For details about calculating both A and g, see [3]. In most cases, both g and its partial derivatives can be calculated analytically. If an appropriate norm of $\vec{i}_e$ is not as small as desired for $\vec{v} = \vec{v}_r$ a new value for $\vec{v}$, say $\vec{v}_{r+1}$, which may reduce the error, can be calculated as $\vec{v}_{r+1} = \vec{v}_r - J^{-1} \times e(\eta, \vec{v}_r)$.

A. Motivation for Introducing η

It is not always possible to guess a good initial value for $\vec{v}$ at a given value of η. If the pair $(\eta, \vec{v})$ is close to a local minimum of e no further minimization by a Newton–Raphson type method may be possible. However, if $\eta = 0$ then $\vec{i}_n = 0$ is obviously a solution. If all partial derivatives of g are continuous at 0, and $\det J \neq 0$, the implicit function theorem guarantees the existence in some neighborhood of the point $(\eta, \vec{v}) = (0, 0)$ of a function $h = (h^0, \cdots, h^{N-1})$ such that $e(\eta, h(\eta)) = 0$. The derivative of h with respect to η is given by

$$\begin{bmatrix} \dfrac{\partial h^0}{\partial \eta} \\ \vdots \\ \dfrac{\partial h^{N-1}}{\partial \eta} \end{bmatrix} = J^{-1} \times A \times v_s. \tag{2}$$

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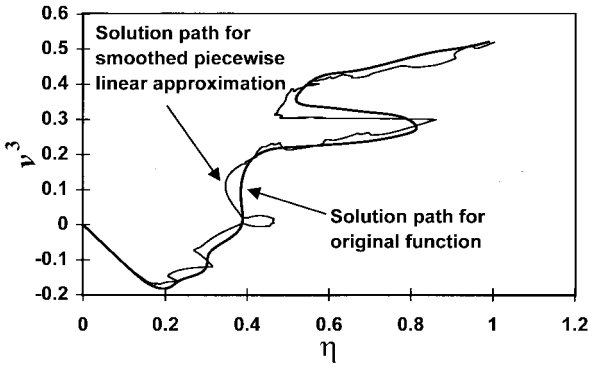

Fig. 2. Graph of points (η, v^3) such that $e(\eta, \vec{v}) = 0$ illustrating the behavior of $e(\eta, \vec{v})$. The solution paths for both the original function and the smoothed piecewise linear approximation are shown. See Fig. 3 for an explanation.

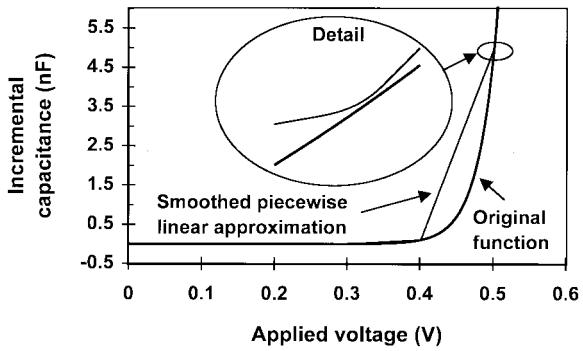

Fig. 3. Original smooth function used to model the incremental diode capacitance together with the smoothed piecewise linear approximation. The results of the analysis for both functions are shown in Fig. 2.

Thus, one can estimate an initial value for $\vec{v}$ at η where η is small as $\eta \times J^{-1} \times A \times \vec{v}_s$ where J is calculated at $(\eta, \vec{v}) = (0, 0)$. If η is kept small enough, then hopefully Newton's method will converge and yield a solution at η. If for each value of η one can calculate a larger value for η, say $\eta + \Delta\eta$, where Newton's method converges and $\Delta\eta$ does not become prohibitively small at some stage, one can continue in this fashion until $\eta = 1$. The function $\eta \times v_s$ where $\eta = 1$ is considered as the function for which a solution is desired, so if the process continues until $\eta = 1$, a solution has been found.

Unfortunately, it is generally not true that one can continue increasing η and keep on finding points $(\eta, \vec{v})$ where the Newton–Raphson method converges. To show why this is not the case, consider the graph of points (η, v^3) such that $e(\eta, \vec{v}) = 0$, as shown in Fig. 2.

The points were obtained for a circuit analyzed by the method outlined in the following section. From Fig. 2 it is clear that as η is increased from 0 to 1, points are encountered where the partial derivative of the local function h^3 with respect to η becomes infinite. At these points, another point where $e(\eta, \vec{v}) = 0$ cannot easily be found by the procedure outlined in the preceding paragraphs.

B. Finding Connected Solutions When One of the Partial Derivatives of the Local Function h Becomes Very Large

Suppose that a solution had been obtained at the point $(\eta_a, \vec{v}_a)$. The conditions that must be satisfied in order that the implicit function theorem guarantees the existence of a local function h in some neighborhood of $(\eta_a, \vec{v}_a)$ such that $e(\eta, h(\eta)) = 0$, are that the partial derivatives $\partial e^k / \partial v^j, 0 \le k, j \le N-1$ evaluated at the point $(\eta_a, \vec{v}_a)$ are all continuous and that the determinant of the matrix J evaluated at the point $(\eta_a, \vec{v}_a)$ is not zero. The first condition can usually be satisfied by being careful with the definition of the function g (e.g., by expressing the nonlinear resistance and capacitance in terms of exponential functions rather than piecewise linear functions). It will be assumed that this condition is satisfied.

Suppose that a point $(\eta_a, \vec{v}_a)$ with $e(\eta_a, \vec{v}_a) = 0$ is encountered where $\det J = 0$. Normally one views η as a parameter and considers $\vec{v}$ as the unknown quantity that must be calculated at a given value of η such that $e(\eta, \vec{v}) = 0$. There is, however, no reason why η should be considered as the independent variable and one can consider a one-to-one differentiable function s such that $(\eta, \vec{v}) = s(y, \vec{w})$. Then, $e(\eta_a, \vec{v}_a) = (e \circ s)(y_a, \vec{w}_a) = 0$. With a suitable choice of s it may be possible to satisfy the criteria for applying the implicit function theorem so that locally a function u exists such that $(e \circ s)(y, u(y)) = 0$. Since the derivative of u can be calculated, one can estimate a new point $(y_b, \vec{w}_e)$ and change the iteration procedure to adjust $\vec{w}$ while keeping y_b fixed to obtain a new solution such that $(e \circ s)(y_b, \vec{w}_b) = 0$.

If one is interested in finding solutions on the same solution branch (i.e. you don't want to jump to a distinct solution branch) and $(\eta_a, \vec{v}_a)$ is a solution, then one wants to have the next solution point satisfy $(\eta_b, \vec{v}_b) = s(y_b, u(y_b))$ where u is the unique local function such that $(e \circ s)(y, u(y)) = 0$ and $(\eta_a, \vec{v}_a) = s(y_a, u(y_a))$. In practice, this involves setting limits on how far the solution point may deviate from the predicted point. The setting of such limits are also desirable to prevent one from missing important features of a particular solution branch such as jump resonances [6].

It is impractical to keep on calculating points $(\eta, \vec{v})$ until $\det J = 0$. It is easier to set a limit on the largest absolute value of any partial derivative appearing in (2). If this limit is exceeded, a suitable function s is the linear function that swaps η with v^k. A good choice of k is the row in (2) with the largest absolute value. As long as the rank of the Jacobian matrix $e'(\eta, \vec{v})$ is N, this produces new variables y and $\vec{w}$ for which the criteria for applying the implicit function theorem are satisfied. New solutions can then be found by changing y as the independent variable and iterating $\vec{w}$ until the partial derivatives in (2) are again within limits, at which point η can once again be used as the independent variable. When switching between η and y as independent variables, the sign of the last increment in the new independent variable is maintained to prevent back tracking to previously determined solution points.

III. Effect of Device Modeling

In the preceding section, care was taken to show that the continuation method is justified by the implicit function theorem. The application of the theorem requires that the error function e is continuously differentiable. This is only a sufficient condition and the question remains how the performance of the method is affected by an error function that is not continuously differentiable. A varactor circuit was analyzed with both a smooth and a piecewise linear approximation for the diode capacitance. The method failed to find the entire solution path when the piecewise linear approximation was used.

To get some insight into the failure, a second experiment was done where a fraction of each section of the piecewise linear approximation was used to create a polynomial fit joining the section smoothly with the next section. This is shown in Fig. 3.

The continuation method managed to trace the entire solution path up to where only 1/40th of each section was used for the polynomial fit. From Fig. 2, it is clear that the solutions for the original smooth function and the smoothed piecewise linear approximation have the same broad characteristics. However, the solution path of the

smoothed piecewise linear approximation has a large number of very sharp bends. In order to negotiate these bends the limits on the difference between the predicted next solution point using the derivative and the actual one found by the Newton–Raphson step had to be set in the order of 0.1%. It required a total of 1676 iterations and 830 s (on a personal computer with a 75-MHz Pentium processor) to trace the entire solution path. For the original smooth function, the percentage tolerance could be set to 5% and it required 101 iterations and 37.5 s to trace the entire solution path.

Note that for the particular circuit there are solutions not on the solution path traced out by the algorithm. A random search at $\eta = 1$ found 14 distinct solutions.

IV. Conclusion

The motivation of the parameter switching continuation method from the point of view of the implicit function theorem shows that a smooth error function forms part of a set of sufficient conditions to implement the algorithm locally. It was shown that the performance of the algorithm is greatly dependent on the smoothness of the error function.

There is no method of ensuring that all solutions at a particular input level have been found. This is inherent in the nature of the implicit function theorem, namely that it locally guarantees the existence of a unique solution, but not globally.

The inability to determine whether all solution paths have been found leaves a gap in the analysis of nonlinear microwave circuits. In the case where the nonlinear elements are limited to nonlinear resistors there is a theory by which it can be shown that under some simple assumptions multiple solutions cannot exist [7]. No such theory does, to the knowledge of the author, exist for dealing with circuits containing combinations of nonlinear resistors and capacitors as is frequently encountered in microwave circuits.

Appendix

In this appendix, the equations necessary to predict new values for $(\eta, \vec{v})$ and to apply a Newton–Raphson method to adjust $\vec{w}$ until $(e \circ s)(y, \vec{w}) = 0$ at a fixed value of y are derived for the case where s is the function that interchanges η with v^k for some k.

Care is taken to determine the requirements for the existence of the various derivatives. One assumes that the matrix A and the function g appearing in (1) are known and that g is continuously differentiable.

From (1) it follows that $e'(\eta, \vec{v}) = [-A \times \vec{v}_s | A + J_g]$where

$$J_g = \begin{bmatrix} \dfrac{\partial g^0}{\partial v^0} & \cdots & \dfrac{\partial g^0}{\partial v^{N-1}} \\ \vdots & & \\ \dfrac{\partial g^{N-1}}{\partial v^0} & \cdots & \dfrac{\partial g^{N-1}}{\partial v^{N-1}} \end{bmatrix}$$

and all partial derivatives are evaluated at $(\eta, \vec{v})$.

Let

$$e'(\eta, \vec{v}) = J_a(\eta, \vec{v}) = [J_a^1(\eta, \vec{v}) | \cdots | J_a^{N+1}(\eta, \vec{v})]$$

where $J_a^i(\eta, \vec{v})$ is the ith column of $J_a(\eta, \vec{v})$.

Define s_j: $\mathcal{R}^{N+1} \rightarrow \mathcal{R}^{N+1}$ so that

$$s_j(x^1, \cdots, x^{N+1}) = (x^{\sigma(1)}, \cdots, x^{\sigma(N+1)})$$

where

$$\sigma(i) = \begin{cases} j & \text{if } i = 1 \\ 1 & \text{if } i = j \\ i & \text{if } i \notin \{1, j\}. \end{cases}$$

Define

$$\begin{aligned} J_{b_j}(y, \vec{w}) &= [J_{b_j}^1(y, \vec{w}) | J_{b_j}^2(y, \vec{w})] \\ &= [J_a^{\sigma(1)}(s_j(y, \vec{w})) | \cdots | J_a^{\sigma N+1)}(s_j(y, \vec{w}))] \end{aligned}$$

where $J_{b_j}^1(y, \vec{w})$ is the first column of $J_{b_j}(y, \vec{w})$. Let

$$\begin{aligned} (y, \vec{w}) = s_j(\eta, \vec{v}) &\Rightarrow (e \circ s_j)'(y, \vec{w}) \\ &= J_{b_j}(y, \vec{w}) = [J_{b_j}^1(y, \vec{w}) | J_{b_j}^2(y, \vec{w})]. \end{aligned} \tag{3}$$

Let $(\eta_a, \vec{v}_a)$ be a solution point, i.e., $e(\eta_a, \vec{v}_a) = 0$. If $\operatorname{rank}(J_a(\eta_a, \vec{v}_a)) = N$ then for at least one choice of j one has $\det(J_{b_j}^2(y_a, \vec{w}_a)) \neq 0$, where $(y_a, \vec{w}_a) = s_j(\eta_a, \vec{v}_a)$. If all partial derivatives appearing in $J_a(\eta_a, \vec{v}_a)$ are continuous, so are all partial derivatives appearing in $J_{b_j}(y_a, \vec{w}_a)$. All criteria for applying the implicit function theorem are thus satisfied. Therefore, there exists an open set U containing y_a, an open set V containing $\vec{w}_a$, and a differentiable function u such that for each y in U there is a unique $u(y)$ in V such that $(e \circ s_j)(y, u(y)) = 0$. By utilizing the fact that u is differentiable and $e \circ s_j$ is identically zero on U, one can determine the derivative of u with respect to y at y_a:

$$u'(y_a) = -[J_{b_j}^2(y_a, \vec{w}_a)]^{-1} \times J_{b_j}^1(y_a, \vec{w}_a). \tag{4}$$

Equation (4) is used to predict the new solution point and (3) is used in the Newton–Raphson step. As can be seen, implementation of the algorithm simply involves exchanging of columns in some Jacobian matrices.

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